Model Updating Using Operational Data

T. Lauwagie[1], J. Guggenberger[2], J. Strobbe[1], E. Dascotte[1]

[1] Dynamic Design Solutions NV
Interleuvenlaan 64, B-3001, Leuven, Belgium
email: **tom.lauwagie@dds.be**

[2] Müller-BBM GmbH
Robert-Koch-Straße 11, D-82152, Planegg, Germany

Abstract

Finite element model updating is a well established method for validating and improving simulation models in structural dynamics. The traditional approach consists of correlating simulation data with the results of an experimental modal analysis (EMA). Usually, EMA data, i.e. resonance frequencies and mode shapes extracted from frequency response functions, is used as reference since it is independent of the applied loads.

However, the operational loads or boundary conditions can change the dynamic behavior of a structure, or make it impossible to perform an experimental modal analysis. In such cases, only operational data can be used as reference data for model updating. Additionally, updating a model using operational data automatically guaranties the validity of the model under the considered operational conditions.

This paper introduces a new updating approach based on Operational Deflection Shapes (ODS) that is able to update the mass, stiffness and damping properties of a structure simultaneously. The proposed method is evaluated by means of a simulated experiment.

1 Introduction

Experimental modal analysis (EMA) [1] is relatively fast and easy to apply. From a set of measured Frequency Response Functions (FRF) it estimates resonance frequencies with a small scatter and mode shapes which reveal global information of the structure. From the analytical side, e.g. using Finite Element Modeling (FEM), linear modal analysis is a fairly simple and well understood process. Furthermore, efficient sensitivity computation algorithms are available for resonant frequencies and mode shapes. This makes EMA-data well-suited for updating FE-models.

However, the use of EMA data also has a number of disadvantages. Linear modal analysis does not depend on external forces. Therefore, EMA data is far from optimal for validation purposes in case the operational loads that change the behavior of the structure. In some situations it might not even be possible to perform an experimental modal analysis due to the environmental conditions under which the structure is operating. Furthermore, it is not possible to update the physical damping properties of a structure using the experimental modal damping ratios of the vibration modes because FE programs use modal damping as input and not as response. The physical damping properties of FE-models can only be update by using the measured FRFs as reference data for the updating.

Operational Modal Analysis (OMA) [1] is an alternative for experimental modal analysis that has become popular within the recent years. The method has mainly been developed by the civil engineering community because artificial excitation is difficult to apply with large structures. By averaging response spectra with random excitation (wind, traffic, etc.) mode shapes, resonance frequencies and damping ratios can be identified at the spectral peaks. Although, OMA overcomes the problems of measuring under operational conditions, a number of issues remain. OMA mode shapes tend to be noisier than EMA mode shapes. As, in most cases, OMA only provides a limited number of modes, the possibilities to cancel out the noise by using multiple responses that are sensitive to the same model parameters are limited. Like EMA-based updating, OMA-based updating cannot tune the damping parameters of the FE-model.

In the EMA case, updating physical damping properties requires a set of experimental FRFs. The operational equivalent of FRFs are operational response spectra. These response spectra can be transformed into a set of Operational Deflection Shapes (ODS), i.e. one deflection shape for every considered frequency line. Therefore, ODS-based updating seems to be an interesting approach to update FE-model using operational data.

2 Updating Using Operational Deflection Shapes (ODS)

Operational deflection shapes contain information on the stiffness, mass and damping properties of the structure. As operational shapes can be measured at any frequency line of interest, they provide a much richer data set than mode shapes. By using a richer data set, the impact of the noise of every individual shape becomes less important. ODS data can be obtained by testing under operational conditions, however, in order to be able to use the ODS data as reference data for FE model updating they have to be computed analytically. To simulate the ODS, the dynamic excitation forces, i.e. the amplitude and phase in function of the excitation frequency, have to be applied to the FE-model. This requires knowledge of the excitation forces, for example from measurements.

Important potential applications of ODS-based model updating are monitoring of vibrating machinery for damage and structural health. Note that updating a model using operational data automatically guaranties the validity of the model under the considered operational conditions.

2.1 Updating Process

The updating cycle starts with the computation of the numerical ODS. For computational efficiency, the ODS are obtained using a modal superposition approach [2], i.e. in a first step the normal modes of the structure are computed, then the ODS are derived from these modes using the applied forces. The numerical ODS are compared with the experimental ODS and the sensitivity of the ODS responses with respect to the updating parameters is computed. Next, the parameter corrections are estimated using a Bayesian [3] parameter estimator:

$$\{\Delta P\} = \left([C_P] + [S]^T [C_R][S]\right)^{-1} [S]^T [C_R]\{\Delta R\} \tag{1}$$

in which {ΔP} are the resulting parameter modifications, {ΔR} are the current response differences, [S] is the sensitivity matrix and [C_R] and [C_P] are diagonal matrices comprising the confidence factor in the responses and parameters, respectively.

The resulting parameter corrections are applied to the FE-model and the responses of the updated model are computed. Note that if the updating parameters do not modify the stiffness or mass matrices, e.g. damping parameters, the normal modes do not change during the updating process. This implies that the FE-model only has to be solved once, leading to a very efficient updating process.

2.2 Response Selection

The ODS-based updating algorithm uses correlation metrics between the numerical and experimental operational shapes as targets. The correlation metrics are computed at the frequency lines of the selected operational shapes.

The following three correlation metrics can be used as updating responses:

- Displacement Assurance Criterion (DAC). The DAC expresses the overall shape correlation between two operational shapes and is defined as:

$$DAC = 100 \frac{\left|\{\psi_a\}^T \{\psi_e\}\right|^2}{\left|\{\psi_a\}^T \{\psi_a\}\right|\left|\{\psi_e\}^T \{\psi_e\}\right|} \tag{2}$$

The target value of a DAC response is 100, i.e. perfect correlation.

- Displacement Scaling Factor (DSF). The DSF expresses the overall amplitude difference between two operational shapes and is defined as

$$DSF = \frac{\left|\{\psi_a\}^T \{\psi_e\}\right|}{\{\psi_e\}^T \{\psi_e\}} \tag{3}$$

The target value of a DSF response is 1.

- Displacement Phase Correlation. The DPC expresses the overall phase correlation between two operational shapes and is defined as

$$DPC = 100\left(1 - \frac{\left|\angle\left(\{\psi_a\}^T \{\psi_e\}\right)\right|}{\pi}\right) \tag{4}$$

in which $\angle$ denotes the phase angle of a complex number. The target value of a DPC response is 100.

2.3 Sensitivity Analysis

A key step in the practical application of ODS-based updating is the computation of the sensitivity coefficients. A differential sensitivity computation approach is fast but requires analytical expressions for the sensitivity coefficients of the DAC, DSF and DPC values. The derivation of such formulas would be a challenging, if not impossible, task. A finite different sensitivity computation approach is easy to implement, but requires one additional FE computation for each considered parameters. This would have a significant impact on the practical applicability of ODS-based updating.

However, by combining the finite difference and differential approach it is possible to create a fast and easy-to-implement algorithm to compute the sensitivity coefficients of the ODS responses. Starting from the equations of equilibrium of a forced harmonic response analysis:

$$[Z]\{\psi\} = \{F\} \tag{5}$$

where $[Z]$ is the dynamic stiffness:

$$[Z] = -\omega^2[M] + i\omega^2[B] + [K] + i[C] \tag{6}$$

The partial derivatives of the ODS can be expressed as:

$$[Z]\frac{\partial\{\psi\}}{\partial p} = \underbrace{\frac{\partial\{F\}}{\partial p}}_{=0} - \frac{\partial[Z]}{\partial p}\{\psi\} \tag{7}$$

In modal space, equation (7) becomes:

$$\frac{\partial\{q\}}{\partial p} = -[z]^{-1}\frac{\partial[z]}{\partial p}\{q\} \tag{8}$$

in which

$$
\begin{aligned}
[z] &= -\omega^2[m] + i\omega^2[b] + [k] + i[c] \\
\frac{\partial[z]}{\partial p} &= -\omega^2\frac{\partial[m]}{\partial p} + i\omega^2\frac{\partial[b]}{\partial p} + \frac{\partial[k]}{\partial p} + i\frac{\partial[c]}{\partial p}
\end{aligned} \tag{9}
$$

where $[m]$, $[k]$, $[b]$ and $[c]$ are the modal mass, stiffness and damping matrices respectively. By considering a finite difference step instead of a partial derivative, equation (8) can be rewritten as:

$$
\{\Delta q\} = -[z]^{-1}[\Delta z]\{q\} \tag{10}
$$

in which $[\Delta z]$ equals

$$
[\Delta z] = [z(p+\Delta p)] - [z(p)] \tag{11}
$$

By transforming the modal displacement vector $\{\Delta q\}$ to the spatial domain, the modification of the ODS by modifying the parameter p with Δp is obtained.

$$
\{\Delta\psi\} = [\Psi]^T\{\Delta q\} \tag{12}
$$

The evaluation of (12) only requires solving the FE-model in modal space, which implies that the evaluation of this expression is very time efficient.

Eventually, the sensitivity of the DAC with respect to a particular model parameter can be expressed as:

$$
\begin{aligned}
\frac{\partial DAC}{\partial p} &\approx \frac{DAC(p+\Delta p) - DAC(p)}{\Delta p} \\
&= \frac{100}{\Delta p}\left(\frac{\left|\{\psi_a + \Delta\psi_a\}^T\{\psi_e\}\right|^2}{\left|\{\psi_a+\Delta\psi_a\}^T\{\psi_a+\Delta\psi_a\}\right|\left|\{\psi_e\}T\{\psi_e\}\right|} - \frac{\left|\{\psi_a\}^T\{\psi_e\}\right|^2}{\left|\{\psi_a\}^T\{\psi_a\}\right|\left|\{\psi_e\}T\{\psi_e\}\right|} \right)
\end{aligned} \tag{13}
$$

in which $\{\Delta\psi\}$ is obtained with expression (12). Note that the DSF and DPC sensitivities can be computed with the same $\{\Delta\psi\}$ vectors used in expression (13).

3 Example

The ODS-based updating method, introduced in the previous paragraph, has been implemented in the FEMtools Model Updating software [4]. The considered updating technique will be illustrated and evaluated by means of an example.

3.1 Set-up

The example considers a small electric motor, mounted on a mass, supported by springs. The springs are attached to a thick base plate which is supported at the corners by polymer blocks. The test set-up is presented in Figure 1.

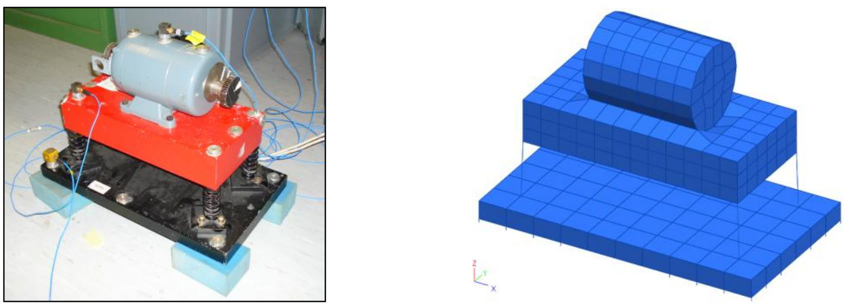

Figure 1: The considered test set-up (left) and the FE-model (right).

To investigate the performance of the new ODS-based updating routine, ‘simulated test data’ is used to update the FE-model. In this way it is possible to verify how close the updated parameter values lie to the ‘correct’ value. The FE-model presented in Figure 1 was used to generate the test data. This model has two types of springs: the support springs and the connection springs. The support springs are 4 sets of 12 springs that model the support of the base plate. The connection springs are 4 springs that model that connection between the base plate and the mass supporting the motor. Table 1 presents the properties of the springs: K_x, K_y and K_z are the stiffness in the X, Y and Z direction while GE is the structural damping value.

	Support spring	Connection springs
K_x	1000	10000
K_y	1000	10000
K_z	10000	100000
GE	0.1	0.01

Table 1: The spring properties of the FE-model.

A set of reference test data was generated by computing the operational deflection shapes between 3 and 35 Hz, with a step of 0.2 Hz, using the spring properties of Table 1. The operational deflection shapes were computed in the response points of the measurement grid shown in Figure 2. In the 9 response points, the acceleration in the X, Y and Z direction was computed. In the excitation point, a unit force was applied in the X, Y and Z direction. The excitation forces had a constant value over the whole frequency range.

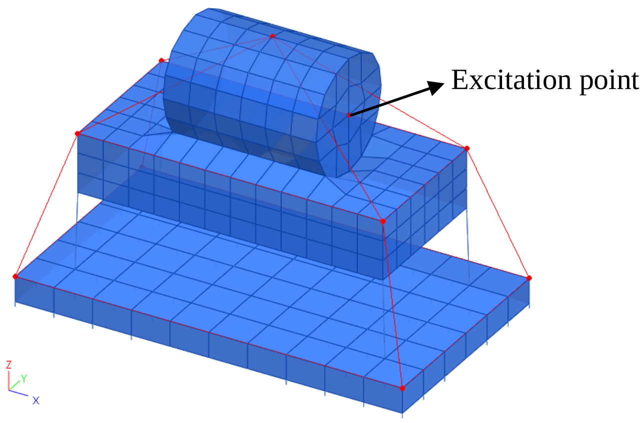

Figure 2: The measurement grid

3.2 ODS-based Updating

To validate the ODS-based updating approach, the spring properties of the FE-model were first altered. Next, the modified FE-model was updated using the operational deflection shapes generated with the reference FE-model. Finally, the updated spring properties were compared with the reference spring properties to verify whether the updating routine was able to identify the correct parameter values.

3.2.1 Test-Case

To evaluate the ODS-based updating approach, both the stiffness and damping properties of the springs were modified. Table 2 presents the modified spring properties used as starting values of this test-case.

Figure 3 show the correlation between the references shapes and the shapes of the FE-model with the modified spring properties.

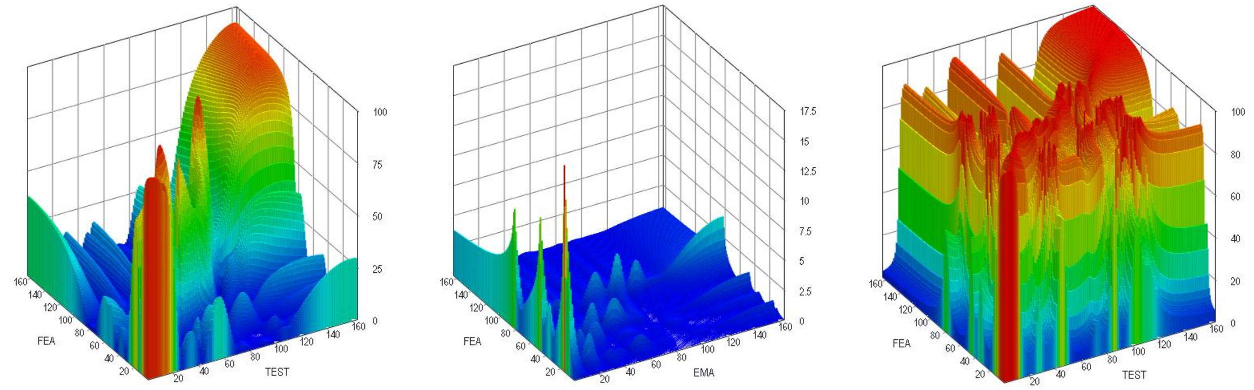

Figure 3: The DAC (left), DSF (middle) and DPC (right) matrices for the starting values of the spring properties.

The full correlation matrices of Figure 3 are not straightforward to interpret due to the abundance of information. Therefore, it makes more sense to only plot the diagonals of those matrices, i.e. the correlation data limited to the corresponding frequency lines.

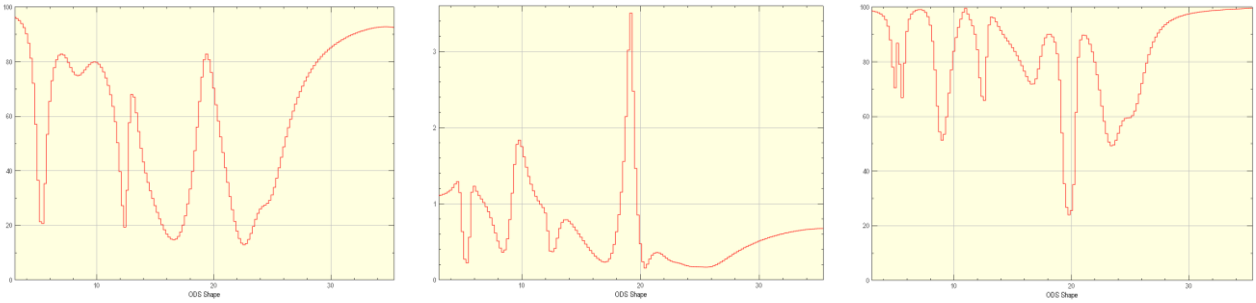

Figure 4: The diagonals of the DAC (left), DSF (middle) and DPC (right) matrices for the starting values of the spring properties.

The model was updated using the Bayesian parameter estimator with uniform confidence factor for all the parameter and responses. During updating, the parameter values converged in a smooth way to stable values. A converged solution was reached after 14 iteration steps. Figure 4 shows the iteration history of an arbitrary response (the DAC value at 10.2 Hz) and an arbitrary parameter (the damping of the connection springs).

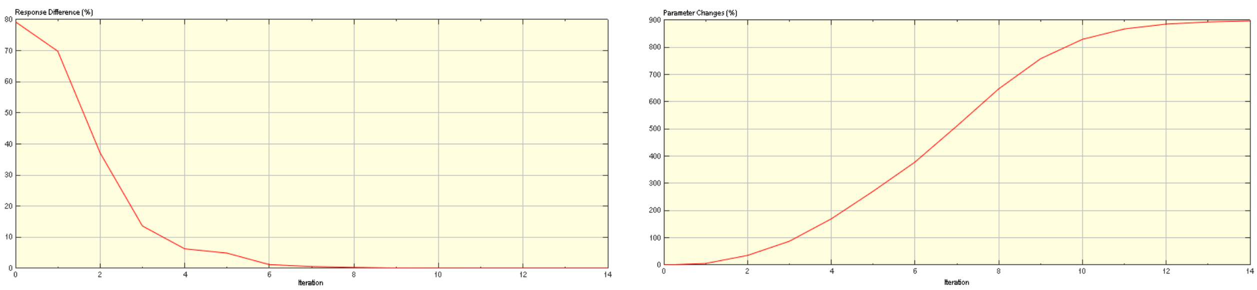

Figure 5: The iteration history of the DAC response at 10.2 Hz (left) and the damping of the connection springs (right).

The goal of the updating process is to get all the DAC and DPC values Figure 4 to 100, and all the DSF values to 1. Figure 6 shows the diagonals of the DAC, DSF and DPC matrices. Note that the DAC and

DPC values are plotted with a Y-scale between 99 and 101, while the DSF is plotted with a Y-scale between 0.99 and 1.01. Therefore, Figure 6 indicates that the final correlation is excellent.

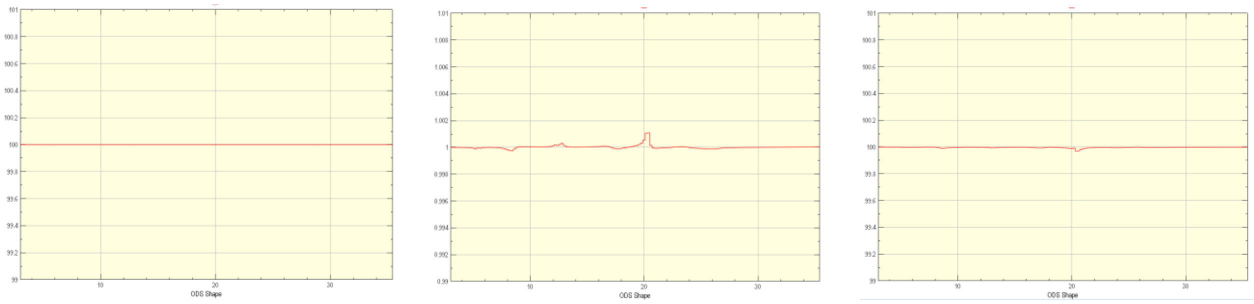

Figure 6: The diagonals of the DAC (left), DSF (middle) and DPC (right) matrices after updating.

The results of the updating are presented in Table 2, which shows that the ODS-based updating routine identified the correct spring properties.

	Supporting blocks				Connection springs			
	Starting	Updated	Reference	Error	Starting	Updated	Reference	Error
K_x	750	1000.0	1000	0.0%	10000	10000.0	10000	0.0%
K_y	1500	1000.0	1000	0.0%	8000	9999.9	10000	0.0%
K_z	5000	10000.0	10000	0.0%	200000	99995.0	100000	0.0%
GE	0.15	0.100	0.1	0.0%	0.001	0.00999	0.01	-0.1%

Table 2: The results of the first test-case.

3.2.2 Impact of the Selected Responses

The updating discussed above used the DAC, DSF and DPC responses for every frequency line. However, it is not clear which responses contribute the information that is required to find the correct solution. In a first phase, a sensitivity analysis was performed. Figure 7 presents the sensitivity coefficients for the DAC, DSF and DPC responses. There appears to be no fundamental differences between the sensitivity coefficients of the three response types.

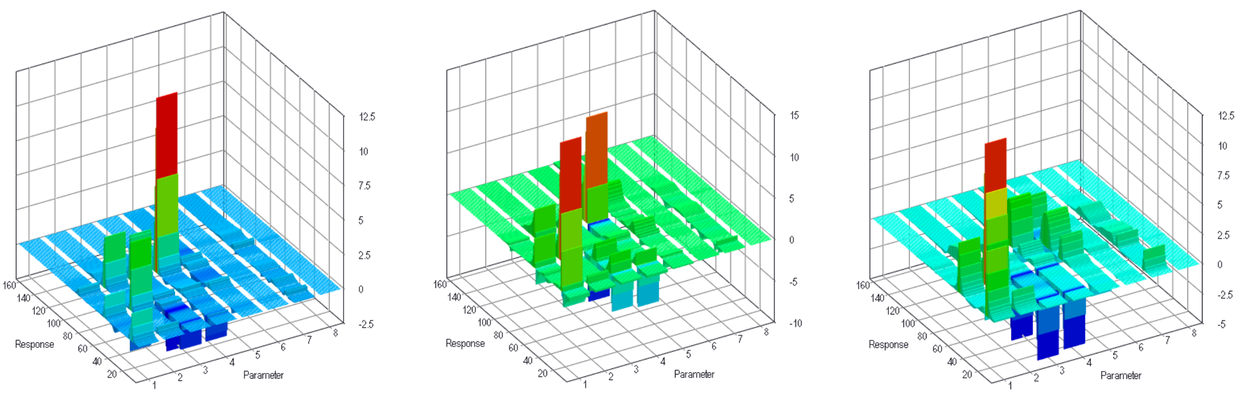

Figure 7: The sensitivity coefficients of the DAC (left), DSF (middle) and DPC (right) with respect to the updating parameters.

In a second phase, the updating problem was solved using all possible response type combinations: only DAC responses, only DSF responses, only DPC responses, DAC and DSF responses, DAC and DPC responses, DSF and DPC responses, and all three responses. The results obtained for each response combination are presented in Table 3. It appears that any response combination that comprises DSF responses provides the correct spring parameters; any combination that does not comprise the DSF responses is not able to provide the correct spring parameters. It is not clear whether this is a general conclusion, or whether this conclusion is only valid for the considered test case. It should be noted that, due to the efficient sensitivity computation, the number of used responses hardly has any impact on the CPU time required to solve the updating problem. Therefore, it is advisable to use all responses types, i.e. DAC, DSF and DPC, unless there is a clear reason not to.

	Support Springs							
	K_x		K_y		K_z		*GE*	
	Value	Error	Value	Error	Value	Error	Value	Error
DAC	1010.3	**1.0%**	996.0	-0.4%	10044.0	0.4%	0.116	**15.6%**
DSF	1000.0	0.0%	1000.0	0.0%	10000.0	0.0%	0.100	0.0%
DPC	946.4	**-5.4%**	1023.9	**2.4%**	9871.6	**-1.3%**	0.109	**8.5%**
DAC-DSF	1000.0	0.0%	1000.0	0.0%	10000.0	0.0%	0.100	0.0%
DAC-DPC	979.2	**-2.1%**	1008.0	0.8%	9818.1	**-1.8%**	0.110	**9.6%**
DSF-DPC	1000.0	0.0%	1000.0	0.0%	10000.0	0.0%	0.100	0.0%
DAC-DSF-DPC	1000.0	0.0%	1000.0	0.0%	10000.0	0.0%	0.100	0.0%

	Connection Springs							
	K_x		K_y		K_z		*GE*	
	Value	Error	Value	Error	Value	Error	Value	Error
DAC	9918.9	-0.8%	9969.6	-0.3%	96450	**-3.6%**	0.00099	**-90.1%**
DSF	10000.0	0.0%	9999.9	0.0%	99996	0.0%	0.00999	-0.1%
DPC	10278.0	**2.8%**	9879.7	**-1.2%**	105010	**5.0%**	0.00161	**-83.9%**
DAC-DSF	10000.0	0.0%	9999.9	0.0%	99996	0.0%	0.00999	-0.1%
DAC-DPC	10131.0	**1.3%**	9949.7	-0.5%	105210	**5.2%**	0.00175	**-82.5%**
DSF-DPC	10000.0	0.0%	9999.9	0.0%	99997	0.0%	0.00999	-0.1%
DAC-DSF-DPC	10000.0	0.0%	9999.9	0.0%	99995	0.0%	0.00999	-0.1%

Table 3: The spring parameter obtained with different response type combinations.

3.3 FRF-based Updating

Because the excitation forces used to generate the operational defection shapes were unit forces, the combination of all the operational shapes provide in fact the FRFs in the measured DOFs. The considered test-case can, therefore, also be solved using FRF-based updating algorithms.

3.3.1 Initial correlation

The FRF-based updating will be performed using the FRF correlation function based approach. The considered responses will be the CSAC and CSF correlation function values [5] between 3 and 35 Hz, with a frequency step of 0.2 Hz. The starting values are presented in Table 4, and are identical to those used in the previous paragraph. Figure 8 shows the CSAC and CSF correlation function for the initial parameter values. Both functions indicate a relative poor initial correlation between the FE-model and test data.

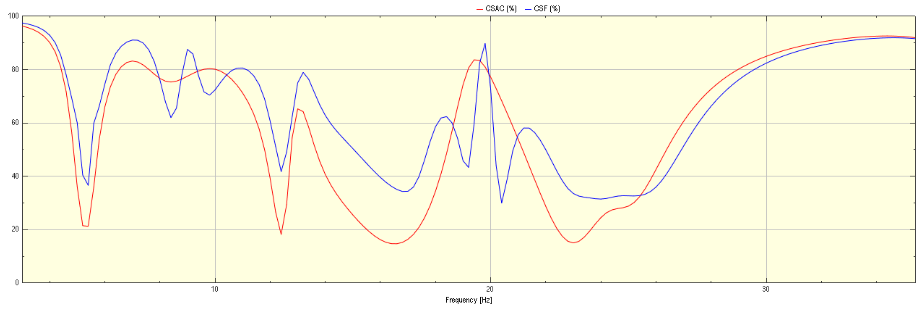

Figure 8: The CSAC (red) and CSF (blue) FRF correlation functions for the initial parameter values.

3.3.2 Updating

The FE-model was updated with the Bayesian parameter estimator using identical confidence values for all the responses and parameters. In a first phase the model parameters seemed to converge to a set of stable values. However, after about 10 iterations, the parameter values stated oscillating and the requested level of convergence was never reached. Eventually the updating procedure was manually aborted after 20 iteration steps. Figure 9 presents the iteration histories of an arbitrary response (CSF value at 8.0 Hz) and an arbitrary updating parameter (vertical stiffness of the connection springs). Both iteration histories clearly exhibit an oscillary behavior.

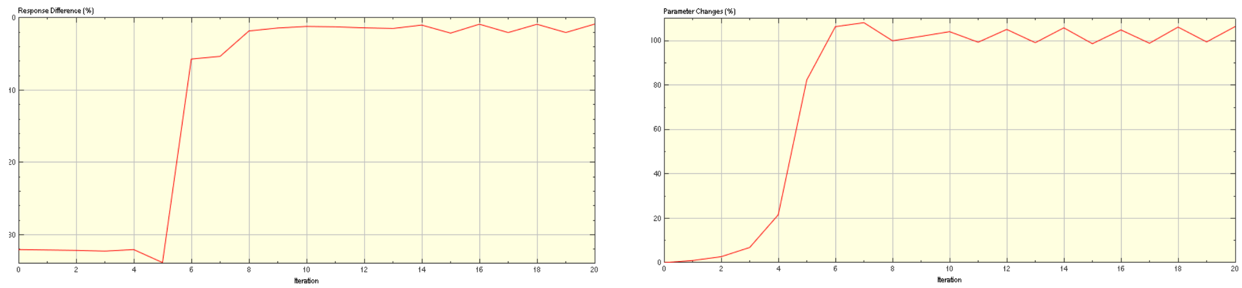

Figure 9: The iteration history of the CSF response at 8.0 Hz (left) and the K_z of the connection springs (right).

The updating provided a significant improvement in the CSAC and SCF values. However, the updating procedure was not able to eliminate all the differences between the simulation and 'test' data, as for a number of frequency lines the CSAC and CSF values did not reach 100.

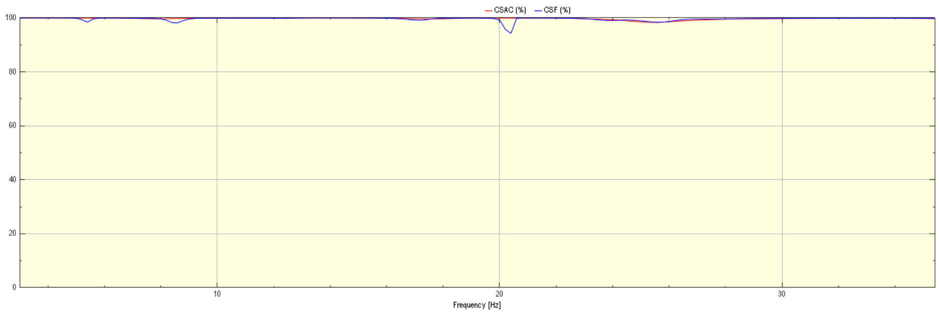

Figure 10: The FRF correlation functions before (top) and after (bottom) updating.

The spring properties identified by the updating routine are presented in Table 4. As the updating routine was no able to reduce all the discrepancies between the simulated and measured responses, it is not a surprise that the correct spring properties were not identified. While the routine provided an acceptable approximation of the spring properties, i.e. error of maximum 6%, the quality of the identified damping properties was poor, i.e. errors up to 99%.

	Supporting blocks				Connection springs			
	Starting	Updated	Reference	Error	Starting	Updated	Reference	Error
K_x	750	1006.7	1000	0.7%	10000	9957.4	10000	-0.4%
K_y	1500	994.7	1000	-0.5%	8000	9984.7	10000	-0.2%
K_z	5000	10320.2	10000	-3.2%	200000	93813.0	100000	-6.2%
GE	0.15	0.125	0.1	24.5%	0.001	0.000118	0.01	-98.8%

Table 4: The results of the first test-case.

3.4 ODS-based versus FRF-based updating

For the considered test-case, the ODS-based updating approach is clearly superior to the FRF-based updating approach. ODS-based updating was capable of retrieving the exact values of the spring properties, while FRF-based updating could only find approximate values for the spring stiffness. The FRF-based updating approach was not able to retrieve the correct damping values. Furthermore, the ODS-based updating routine required significantly less time to solve the problem than the FRF-based updating technique, mainly because the sensitivity analysis was much faster in case of ODS-based updating. This implies that the suggested ODS-based updating approach is a promising approach to solve updating problems with a high number of parameters and responses.

Note that, for the considered example, the FRF-based and ODS-based updating routines used the exact same data but in a different way: the FRF-based updating used for each DOF the full frequency function, while the ODS-based updating used the FRF values of all the DOF for each particular frequency line. The differences in the results obtained are thus entirely due by the way the data was processed by the two routines, not due to differences in the reference data.

4 Conclusions

This paper introduced a new ODS-based updating approach. For the considered test-case the ODS-based updating approach was able to identify the correct model parameters, while the FRF-based approach only managed to get approximate values for the stiffness properties. ODS-based updating also converged to the solution in a stable way, while the FRF-based updating approach suffered from instabilities. ODS-based updating also required a much lower computational effort to update the model than the FRF-based approach. After this first evaluation, the suggested approached thus proved to be a promising updating technique.

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